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Global and local structures of oscillatory bifurcation diagrams

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1 Introduction

We first consider the following example of nonlinear eigenvalue problems

$$-u''(t) = \lambda(u(t) + g(u(t))), \quad t \in I =: (-1, 1), \quad (1.1)$$

$$u(t) > 0, \quad t \in I, \quad (1.2)$$

$$u(-1) = u(1) = 0. \quad (1.3)$$

Here, $g(u) := \frac{1}{2} \sin^k u(t)$, $k \geq 1$ is a given integer, and $\lambda > 0$ is a bifurcation parameter. We know from [15] that the solution set of (1.1)–(1.3) consists of the set

$$Q := \{(\lambda(k, \alpha), u_\alpha) \mid \text{sol. of (1.1)–(1.3) with } \|u_\alpha\|_\infty = \alpha\} \subset \mathbb{R}_+ \times C^2(\bar{I}).$$

Indeed, in this case, for any given $\alpha > 0$, there exists a unique solution pair (λ, u_α) of (1.1)–(1.3) with $\alpha = \|u_\alpha\|_\infty$ and λ is parameterized by α . So we write as $\lambda = \lambda(k, \alpha)$. If we consider the asymptotic behavior of $\lambda(k, \alpha)$ as $\alpha \rightarrow \infty$, then it seems clear that

$$\lambda(k, \alpha) \rightarrow \frac{\pi^2}{4}. \quad (1.4)$$

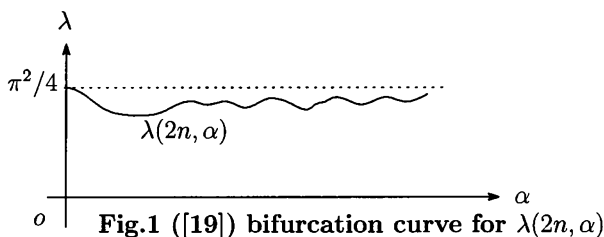
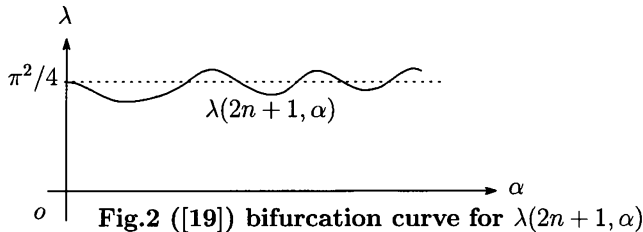


Fig.1 ([19]) bifurcation curve for $\lambda(2n, \alpha)$

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So it is natural to expect that the rate of convergence of $\lambda(2n, \alpha)$ to $\pi^2/4$ as $\alpha \rightarrow \infty$ is the same as that of $\lambda(2n+1, \alpha)$. However, we will find that the following formula holds.

$$|\lambda(2n_1+1, \alpha) - \pi^2/4| \ll |\lambda(2n_2, \alpha) - \pi^2/4| \rightarrow 0, \quad (1.5)$$

where $n_1 \geq 1$ and $n_2 \geq 1$ are arbitrary given integers. To show (1.5), we calculate the asymptotic behavior of $\lambda(k, \alpha)$ precisely.

Theorem 1.1 ([19]). (i) Let $k = 2n$ ($n \geq 1$). Then as $\alpha \rightarrow \infty$

$$\begin{aligned} \lambda(2n, \alpha) &= \frac{\pi^2}{4} - \frac{\pi}{2^{2n+1}\alpha} \binom{2n}{n} - \frac{\pi^{3/2}}{2^{2n+1}\alpha^{3/2}} \sum_{r=0}^{n-1} (-1)^{n-r} \binom{2n}{r} \\ &\quad \times \frac{1}{\sqrt{n-r}} \sin \left((2n-2r)\alpha + \frac{\pi}{4} \right) + O(\alpha^{-2}). \end{aligned} \quad (1.6)$$

(ii) Let $k = 2n+1$ ($n \geq 0$). Then as $\alpha \rightarrow \infty$

$$\begin{aligned} \lambda(2n+1, \alpha) &= \frac{\pi^2}{4} - \frac{\pi^{3/2}}{2^{2n+1}\alpha^{3/2}} \sum_{r=0}^n (-1)^{n+r} \binom{2n+1}{r} \\ &\quad \times \sqrt{\frac{1}{2(2n-2r+1)}} \sin \left((2n-2r+1)\alpha - \frac{1}{4}\pi \right) \\ &\quad + O(\alpha^{-2}). \end{aligned} \quad (1.7)$$

We consider why this kind of difference between (1.6) and (1.7) occurs in the next section.

2 General results

The purpose in this section is to show the reason why such kind of difference between (1.6) and (1.7) occurs. We consider (1.1)–(1.3) and we assume that $g(u)$ satisfies the following conditions.

(A.1) $g(u) \in C^1(\mathbb{R})$ and $u + g(u) > 0$ for $u > 0$.

(A.2) $g(u + 2\pi) = g(u)$ for $u \in \mathbb{R}$.

Then we know from [15] that there exists a unique solution pair (λ, u_α) of (1.1)–(1.3) with $\alpha = \|u_\alpha\|_\infty$ for any given $\alpha > 0$ under the condition (A.1). Besides, λ is parameterized by α as $\lambda(\alpha)$. Moreover, $\lambda(\alpha)$ is a continuous function of $\alpha > 0$. Then it is convenient for us to write $\lambda = \lambda(g, \alpha)$, since λ also depends on g . We note that the Fourier series of g converges uniformly to g under the conditions (A.1) and (A.2).

Now, we introduce the notion of (OP).

(OP) $\lambda(g, \alpha) \rightarrow \pi^2/4$ as $\alpha \rightarrow \infty$, and it intersects the line $\lambda = \pi^2/4$ infinitely many times for $\alpha \gg 1$.

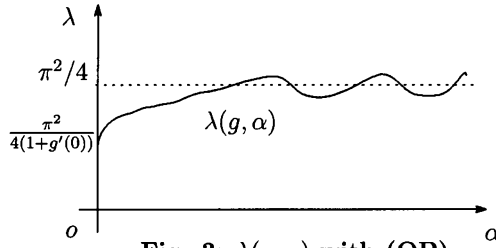


Fig. 3: $\lambda(g, \alpha)$ with (OP)

Since $g(u)$ is bounded in \mathbb{R} by (A.2), it is clear that $\lambda(g, \alpha) \rightarrow \pi^2/4$ as $\alpha \rightarrow \infty$. Therefore, the essential point is to find the condition whether $\lambda(g, \alpha)$ intersects the line $\lambda = \pi^2/4$ infinitely many times for $\alpha \gg 1$. By Theorem 1.1, if $g(u) = \frac{1}{2} \sin^{2n+1}(u)$, then (OP) holds. On the other hand, if $g(u) = \frac{1}{2} \sin^{2n}(u)$, then (OP) does not hold. The purpose here is to find a simple condition, from which we understand whether $\lambda(g, \alpha)$ satisfies (OP) or not immediately.

Now we state our main results.

Theorem 2.1 ([20]). *Assume that $g(u)$ satisfies (A.1)–(A.2). Then as $\alpha \rightarrow \infty$,*

$$\lambda(g, \alpha) = \frac{\pi^2}{4} - \frac{\pi a_0}{2\alpha} - \frac{1}{\alpha} \sqrt{\frac{\pi}{2\alpha}} \sum_{n=1}^{\infty} \frac{c_n}{n^{3/2}} + O(\alpha^{-2}), \quad (2.1)$$

where

$$a_0 := \frac{1}{\pi} \int_{-\pi}^{\pi} g(\theta) d\theta, \quad (2.2)$$

$$c_n := \int_{-\pi}^{\pi} g'(\theta) \cos\left(n(\theta - \alpha) + \frac{3}{4}\pi\right) d\theta, \quad (n \in \mathbb{N}). \quad (2.3)$$

As a corollary of Theorem 2.1, we get an interesting result for the asymptotic behavior of $\lambda(g, \alpha)$.

Corollary 2.2 ([20]). *Assume that $g(u)$ satisfies (A.1)–(A.2). If $a_0 \neq 0$, then $\lambda(g, \alpha)$ does not satisfy (OP).*

By Corollary 2.2, we understand immediately the reason that, in the case of $g(u) = \frac{1}{2} \sin^{2n+1}(u)$, (OP) holds, and in the case of $g(u) = \frac{1}{2} \sin^{2n}(u)$, (OP) does not hold.

The method to study the local behavior of $\lambda(g, \alpha)$ has been already obtained in [17, 18], because the time-map method and Taylor expansion work very well to study the local structure of $\lambda(g, \alpha)$.

Theorem 2.3 ([20]). *Assume (A.1)–(A.2). Furthermore, assume that $g \in C^2$ near $u = 0$. (i) Assume that $g(0) \neq 0$. Then as $\alpha \rightarrow 0$,*

$$\lambda(g, \alpha) = \frac{2\alpha}{g(0)} \{1 + A_1\alpha + A_2\alpha^2 + o(\alpha^2)\}, \quad (2.4)$$

where

$$A_1 = -\frac{5}{6g(0)}(1 + g'(0)), \quad A_2 = \frac{32}{45} \frac{(1 + g'(0))^2}{g(0)^2} - \frac{11}{30} \frac{g''(0)}{g(0)}. \quad (2.5)$$

(ii) Assume that $g(0) = 0$ and $g'(0) > -1$. Then as $\alpha \rightarrow 0$,

$$\lambda(g, \alpha) = \frac{1}{1 + g'(0)} \left(\frac{\pi^2}{4} - \frac{\pi g''(0)}{3(1 + g'(0))} \alpha + o(\alpha) \right). \quad (2.6)$$

3 Global behavior of $\lambda(g, \alpha)$

The proof of Theorem 2.1 is given by the combination of time-map method, Fourier expansion and the asymptotic formulas for some special functions. The proof is given by several steps. In this section, let $\alpha \gg 1$. For simplicity, we write $\lambda = \lambda(g, \alpha)$. Moreover, we denote by C the various positive constants independent of α . Let

$$G(u) := \int_0^u g(s) ds. \quad (3.1)$$

We know that if $(u_\alpha, \lambda) \in C^2(\bar{I}) \times \mathbb{R}_+$ satisfies (1.1)–(1.3), then the following properties hold.

$$u_\alpha(t) = u_\alpha(-t), \quad 0 \leq t \leq 1, \quad (3.2)$$

$$u_\alpha(0) = \max_{-1 \leq t \leq 1} u_\alpha(t) = \alpha, \quad (3.3)$$

$$u'_\alpha(t) > 0, \quad -1 < t < 0. \quad (3.4)$$

Step 1. The well known time-map (3.7) below is constructed as follows. By (1.1),

$$\{u''_\alpha(t) + \lambda(u_\alpha(t) + g(u_\alpha(t)))\} u'_\alpha(t) = 0.$$

By this and putting $t = 0$, we have

$$\frac{1}{2}u'_\alpha(t)^2 + \lambda \left(\frac{1}{2}u_\alpha(t)^2 + G(u_\alpha(t)) \right) = \text{constant} = \lambda \left(\frac{1}{2}\alpha^2 + G(\alpha) \right).$$

By this and (3.4), for $-1 \leq t \leq 0$, we have

$$u'_\alpha(t) = \sqrt{\lambda} \sqrt{\alpha^2 - u_\alpha(t)^2 + 2(G(\alpha) - G(u_\alpha(t)))}. \quad (3.5)$$

It is clear from (A.2) that $|g(u)| \leq C$ for $u \in \mathbb{R}$. Then for $0 \leq s \leq 1$,

$$\left| \frac{G(\alpha) - G(\alpha s)}{\alpha^2(1 - s^2)} \right| = \left| \frac{\int_{\alpha s}^{\alpha} g(t) dt}{\alpha^2(1 - s^2)} \right| \leq \frac{C\alpha(1 - s)}{\alpha^2(1 - s^2)} \leq C\alpha^{-1}. \quad (3.6)$$

By (3.5), (3.6), putting $s := u_\alpha(t)/\alpha$ and Taylor expansion, we have

$$\begin{aligned} \sqrt{\lambda} &= \int_{-1}^0 \frac{u'_\alpha(t)}{\sqrt{\alpha^2 - u_\alpha(t)^2 + 2(G(\alpha) - G(u_\alpha(t)))}} dt \\ &= \int_0^1 \frac{1}{\sqrt{1 - s^2 + 2(G(\alpha) - G(\alpha s))/\alpha^2}} ds \\ &= \int_0^1 \frac{1}{\sqrt{1 - s^2}} \frac{1}{\sqrt{1 + 2(G(\alpha) - G(\alpha s))/(\alpha^2(1 - s^2))}} ds \\ &= \int_0^1 \frac{1}{\sqrt{1 - s^2}} \left\{ 1 - \frac{G(\alpha) - G(\alpha s)}{\alpha^2(1 - s^2)} + O(\alpha^{-2}) \right\} ds \\ &:= \frac{\pi}{2} - \frac{1}{\alpha^2} K(\alpha) + O(\alpha^{-2}). \end{aligned} \quad (3.7)$$

Here,

$$K(\alpha) := \int_0^1 \frac{G(\alpha) - G(\alpha s)}{(1 - s^2)^{3/2}} ds. \quad (3.8)$$

Step 2. We calculate $K(\alpha)$ by the asymptotic formulas for several special functions as $\alpha \rightarrow \infty$. We know that under the conditions (A.1)–(A.2),

$$g(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad (3.9)$$

holds for $x \in \mathbb{R}$ and the right hand side of (3.9) converges to $g(x)$ uniformly on \mathbb{R} . Here,

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} g(\theta) \cos n\theta d\theta = -\frac{1}{n\pi} \int_{-\pi}^{\pi} g'(\theta) \sin n\theta d\theta \\ &:= -\frac{1}{n} \tilde{a}_n \quad (n \in \mathbb{N}_0), \end{aligned} \quad (3.10)$$

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} g(\theta) \sin n\theta d\theta = \frac{1}{n\pi} \int_{-\pi}^{\pi} g'(\theta) \cos n\theta d\theta \\ &:= \frac{1}{n} \tilde{b}_n \quad (n \in \mathbb{N}). \end{aligned} \quad (3.11)$$

Step 3.

Lemma 3.1 ([20]). As $\alpha \rightarrow \infty$,

$$K(\alpha) = \frac{1}{2}a_0\alpha + \frac{1}{\pi}\sqrt{\frac{\pi\alpha}{2}} \sum_{n=1}^{\infty} \frac{c_n}{n^{3/2}} + O(\alpha^{-1/2}). \quad (3.12)$$

Proof. We put $s = \sin \theta$ in (3.8). Then we obtain

$$K(\alpha) = \alpha \int_0^{\pi/2} g(\alpha \sin \theta) \sin \theta d\theta. \quad (3.13)$$

We use here the integration by parts and l'Hôpital's rule. For $n = \mathbb{N}$, let

$$U_n := \int_0^{\pi/2} \cos(n\alpha \sin \theta) \sin \theta d\theta, \quad (3.14)$$

$$V_n := \int_0^{\pi/2} \sin(n\alpha \sin \theta) \sin \theta d\theta. \quad (3.15)$$

By (3.13)–(3.15),

$$\begin{aligned} K(\alpha) &= \alpha \int_0^{\pi/2} g(\alpha \sin \theta) \sin \theta d\theta \\ &= \alpha \int_0^{\pi/2} \left\{ \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(n\alpha \sin \theta) \right. \\ &\quad \left. + \sum_{n=1}^{\infty} b_n \sin(n\alpha \sin \theta) \right\} \sin \theta d\theta \\ &= \alpha \left\{ \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \int_0^{\pi/2} \cos(n\alpha \sin \theta) \sin \theta d\theta \right. \\ &\quad \left. + \sum_{n=1}^{\infty} b_n \int_0^{\pi/2} \sin(n\alpha \sin \theta) \sin \theta d\theta \right\} \\ &= \alpha \left\{ \frac{1}{2}a_0 - \sum_{n=1}^{\infty} \frac{1}{n} \tilde{a}_n U_n + \sum_{n=1}^{\infty} \frac{1}{n} \tilde{b}_n V_n \right\}. \end{aligned} \quad (3.16)$$

Put $\theta = \pi/2 - \phi$ in (3.22). Then by (3.9)–(3.12), (3.14), (3.15) and [9, p.425], we obtain

$$\begin{aligned} U_n &= \int_0^{\pi/2} \cos(n\alpha \cos \phi) \cos \phi d\phi \\ &= \frac{\pi}{4} (\mathbf{E}_1(n\alpha) - \mathbf{E}_{-1}(n\alpha)) \\ &= \frac{\pi}{4} (-Y_1(n\alpha) + Y_{-1}(n\alpha) + O((n\alpha)^{-2})) \\ &= \frac{\pi}{4} \left(-\sqrt{\frac{2}{n\pi\alpha}} \sin \left(n\alpha - \frac{3}{4}\pi \right) + \sqrt{\frac{2}{n\pi\alpha}} \sin \left(n\alpha + \frac{1}{4}\pi \right) \right) \end{aligned} \quad (3.17)$$

$$\begin{aligned}
& + O((n\alpha)^{-3/2}) \\
& = -\sqrt{\frac{\pi}{2n\alpha}} \sin\left(n\alpha - \frac{3}{4}\pi\right) + O((n\alpha)^{-3/2}).
\end{aligned}$$

Here, $\mathbf{E}_\nu(z)$ are Weber functions and $Y_\nu(z)$ are Neumann functions. Moreover,

$$\begin{aligned}
V_n &= \int_0^{\pi/2} \sin(n\alpha \cos \phi) \cos \phi \, d\phi \\
&= \frac{\pi}{4} \{ \mathbf{J}_1(n\alpha) - \mathbf{J}_{-1}(n\alpha) \} \\
&= \frac{\pi}{4} \{ J_1(n\alpha) - J_{-1}(n\alpha) \} \\
&= \frac{\pi}{4} \left\{ \sqrt{\frac{2}{n\pi\alpha}} \cos\left(n\alpha - \frac{3}{4}\pi\right) - \sqrt{\frac{2}{n\pi\alpha}} \cos\left(n\alpha + \frac{1}{4}\pi\right) \right\} \\
&\quad + O((n\alpha)^{-3/2}) \\
&= \sqrt{\frac{\pi}{2n\alpha}} \cos\left(n\alpha - \frac{3}{4}\pi\right) + O((n\alpha)^{-3/2}).
\end{aligned} \tag{3.18}$$

Here, $\mathbf{J}_\nu(z)$ are Anger functions and $J_\nu(z)$ are Bessel functions) By (3.14)–(3.18), we obtain

$$\begin{aligned}
K(\alpha) &= \alpha \left\{ \frac{1}{2}a_0 + \sqrt{\frac{\pi}{2\alpha}} \sum_{n=1}^{\infty} \left(\tilde{a}_n \sin\left(n\alpha - \frac{3}{4}\pi\right) \right. \right. \\
&\quad \left. \left. + \tilde{b}_n \cos\left(n\alpha - \frac{3}{4}\pi\right) \right) \frac{1}{n^{3/2}} \right\} \\
&\quad + O\left(\alpha^{-1/2} \sum_{n=1}^{\infty} \frac{1}{n^{5/2}}\right) \\
&= \alpha \left\{ \frac{1}{2}a_0 + \frac{1}{\pi} \sqrt{\frac{\pi}{2\alpha}} \sum_{n=1}^{\infty} \frac{c_n}{n^{3/2}} \right\} + O(\alpha^{-1/2}).
\end{aligned}$$

Thus the proof is complete. ■

By (3.7) and Lemma 3.1, we obtain Theorem 2.1. ■

We introduce the Special functions and their asymptotic behavior here. For $z \gg 1$, we have (cf. [9, p. 929, p. 958])

$$\begin{aligned}
J_1(z) &= \sqrt{\frac{2}{\pi z}} \left\{ [1 + R_1] \cos\left(z - \frac{3}{4}\pi\right) \right. \\
&\quad \left. - \left[\frac{1}{2z} \frac{\Gamma(\frac{5}{2})}{\Gamma(\frac{1}{2})} + R_2 \right] \sin\left(z - \frac{3}{4}\pi\right) \right\},
\end{aligned} \tag{3.19}$$

$$J_{-1}(z) = \sqrt{\frac{2}{\pi z}} \left\{ [1 + R_1] \cos\left(z + \frac{1}{4}\pi\right) \right\}$$

$$- \left[\frac{1}{2z} \frac{\Gamma(\frac{5}{2})}{\Gamma(\frac{1}{2})} + R_2 \right] \sin \left(z + \frac{1}{4}\pi \right) \Bigg\}, \quad (3.20)$$

$$Y_1(z) = \sqrt{\frac{2}{\pi z}} \left\{ [1 + R_1] \sin \left(z - \frac{3}{4}\pi \right) + \left[\frac{1}{2z} \frac{\Gamma(\frac{5}{2})}{\Gamma(\frac{1}{2})} + R_2 \right] \cos \left(z - \frac{3}{4}\pi \right) \right\}, \quad (3.21)$$

$$Y_{-1}(z) = \sqrt{\frac{2}{\pi z}} \left\{ [1 + R_1] \sin \left(z + \frac{1}{4}\pi \right) + \left[\frac{1}{2z} \frac{\Gamma(\frac{5}{2})}{\Gamma(\frac{1}{2})} + R_2 \right] \cos \left(z + \frac{1}{4}\pi \right) \right\}, \quad (3.22)$$

where

$$|R_1| < \left| \frac{\Gamma(\frac{7}{2})}{8\Gamma(-\frac{1}{2})z^2} \right|, \quad |R_2| < \left| \frac{\Gamma(\frac{9}{2})}{48\Gamma(-\frac{3}{2})z^3} \right|, \quad (3.23)$$

$$\mathbf{J}_{\pm 1}(z) = J_{\pm 1}(z), \quad (3.24)$$

$$\mathbf{E}_{\pm 1}(z) = -Y_{\pm 1}(z) \mp \frac{2}{\pi z^2} + O(z^{-4}). \quad (3.25)$$

4 Special case

Finally, we are interested in the case $g(u) = \sin \sqrt{u}$. In this case, the equation (1.1)–(1.3) has been proposed in Cheng [5] as a model problem which has arbitrary many solutions near $\lambda = \pi^2/4$.

Theorem 4.0.([5]) *Let $g_1(u) = \sin \sqrt{u}$ ($u \geq 0$). Then for any integer $r \geq 1$, there is $\delta > 0$ such that if $\lambda \in (\lambda_1 - \delta, \lambda_1 + \delta)$, then (1.1)–(1.3) has at least r distinct solutions.*

Theorem 4.0 gives us the information about the solution set of (1.1)–(1.3), and we expect that $\lambda(g_1, \alpha)$ satisfies (OP). The purpose here is to prove the expectation above is valid.

Theorem 4.1 ([21]). *Let $g(u) = g_1(u) = \sin \sqrt{u}$. Then as $\alpha \rightarrow \infty$,*

$$\lambda(g_1, \alpha) = \frac{\pi^2}{4} - \pi^{3/2} \alpha^{-5/4} \cos \left(\sqrt{\alpha} - \frac{3}{4}\pi \right) + o(\alpha^{-5/4}). \quad (4.1)$$

We next give the asymptotic behavior of $\lambda(g_1, \alpha)$ as $\alpha \rightarrow 0$.

Theorem 4.2 ([21]). *Let $g(u) = g_1(u) = \sin \sqrt{u}$.*

(i) *As $\alpha \rightarrow 0$, the following asymptotic formula for $\lambda(g_1, \alpha)$ holds:*

$$\lambda(g_1, \alpha) = \frac{3}{4} C_1^2 \sqrt{\alpha} + \frac{3}{2} C_1 C_2 \alpha + O(\alpha^{3/2}), \quad (4.2)$$

where

$$C_1 := \int_0^1 \frac{1}{\sqrt{1-s^{3/2}}} ds, \quad C_2 := -\frac{3}{8} \int_0^1 \frac{1-s^2}{\sqrt{1-s^{3/2}}} ds. \quad (4.3)$$

(ii) Let v_0 be a unique classical solution of the following equation

$$-v_0''(t) = \frac{3}{4} C_1^2 \sqrt{v_0(t)}, \quad t \in I, \quad (4.4)$$

$$v_0(t) > 0, \quad t \in I, \quad (4.5)$$

$$v_0(-1) = v_0(1) = 0. \quad (4.6)$$

Furthermore, let $v_\alpha(t) := u_\alpha(t)/\alpha$. Then $v_\alpha \rightarrow v_0$ in $C^2(I)$ as $\alpha \rightarrow 0$.

The proofs also depend on time-map method.

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